

Activity A

5.2 Using Data to Write a Linear Equation

MATERIALS • 90 centimeters of rope • meter stick

QUESTION How can you use data from an experiment to write a linear equation?

EXPLORE Write a linear equation

STEP 1 Form a group

Work with two other people. Choose one person to tie knots in the rope, one person to measure the rope, and one person to record the results.

STEP 2 Measure and record

Measure the length of the rope in centimeters. This is the length of the rope when the number of knots is zero. Record the length in the table below.

| Number of knots | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------------------------|---|---|---|---|---|---|---|---|---|
| Length of rope (centimeters) | | | | | | | | | |

STEP 3 Tie knots, measure, and record

Tie a knot in the rope. Measure the length of the rope after the knot is tied and record this length in the table.

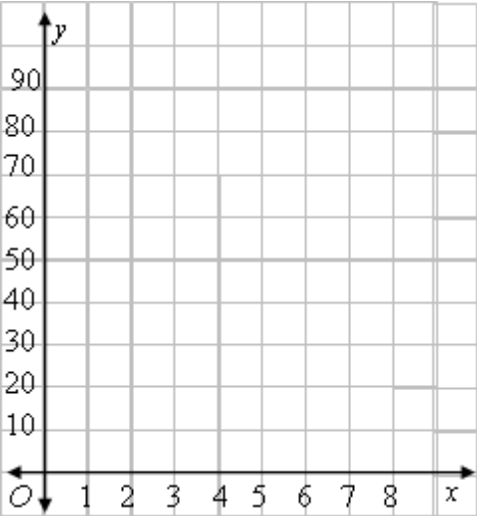
STEP 4 Complete the table

Repeat Step 3 until you complete the table. Tie each knot the same tightness as the others. You will have eight knots in your rope.

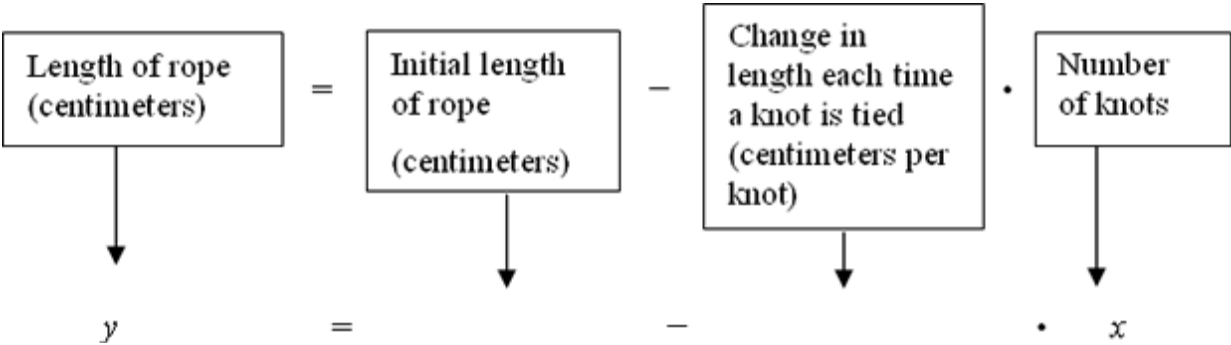
DRAW CONCLUSIONS Use your table of data to complete these exercises

1. As the number of knots increases, does the length of rope *increase* or *decrease*?
2. By about how many centimeters does the length of the rope change from zero knots to one knot? one knot to two knots? Two knots to three knots? Three knots to four knots? By about how many centimeters does the length of the rope change each time a knot is tied?
3. Let x be the number of knots and let y be the length of rope (in centimeters). Write ordered pairs using the data in the table. For example, the first ordered pair may be $(0, 90)$. What does the x -coordinate represent? What does the y -coordinate represent?

4. Use the ordered pairs you wrote in Exercise 3 to make a scatter plot.
 - a. Do you observe any pattern in the scatter plot? *Explain*.
 - b. What is the scale on the x -axis of the graph? Why was that scale chosen?
 - c. What is the scale on the y -axis of the graph? Why was that scale chosen?
 - d. What does the y -intercept represent?



5. Use the verbal model below to help you write an equation that represents this situation.



6. Use the equation you wrote in Exercise 5 to estimate the length of the rope after 10 knots have been tied.
7. Tie two more knots in the rope and measure the length of the rope. How does the length compare with your estimate from Exercise 6?
8. *Compare* your equation and scatter plot with other groups in your class. Do all groups have the same slope? the same y -intercept? the same equation? *Explain*.

Answer Key A

EXPLORE

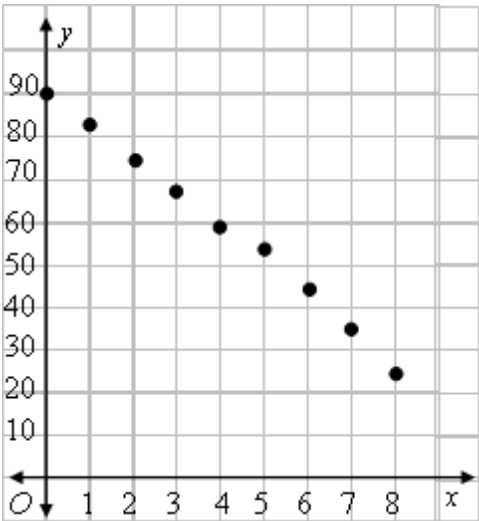
STEP 2 Answers may vary. An example is given.

| Number of knots | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------------------|----|----|----|----|----|----|----|----|----|
| Length of rope (centimeters) | 90 | 82 | 75 | 68 | 59 | 52 | 44 | 36 | 23 |

DRAW CONCLUSIONS

1. decrease
2. Answers may vary. In the example given: 8, 7, 7, 9; the length of the rope decreases by about 8 cm each time a knot is tied.
3. Answers may vary. Check students table of values with their ordered pairs. In the example given: (0, 90), (1, 82), (2, 75), (3, 68), (4, 59), (5, 52), (6, 44), (7, 36), (8, 23).

4. Answers may vary. In the example given:
 - a. The scatter plot appears to be linear. The length of the rope is decreasing.
 - b. The scale on the x -axis is 1. Because the x -axis represents the number of knots, it goes from 0 to 8.
 - c. The scale on the y -axis is 10. The y -axis represents the length of the rope, which starts at 90 and decreases, so increments of 10 make sense.
 - d. It represents the length of the rope before any knots are tied.



5. Answers may vary. In the example given: $y = 90 - 8x$
6. Answers may vary. In the example given: 10 cm. Check to see if students substituted 10 into the equation for x .
7. Answers may vary. For example, 9 cm. The estimate in Exercise 6 is very close to the actual length of the rope.
8. Answers will vary. In general, all groups will have about the same slope. A group that tied loose knots will have a scatter plot with a steeper slope. A group with tight knots will have a scatter plot with a less steep slope. All groups started with the same length of rope (90 cm), so the y -intercepts should all be at (0, 90).

Teacher Notes

ACTIVITY PREPARATION AND MATERIALS

- You will need to cut a piece of rope for each group of 3 students doing the activity. You will not be able to reuse the rope from one class to another because of the knots, so be sure to cut enough rope for all the classes doing the activity.
- Clothesline or very thick twine are a good choice for the rope. Anything less thick than clothesline will require very precise measuring.
- The science department might be a good source for meter sticks.
- If you cannot find meter sticks, you can use yard sticks that are also marked in centimeters.

ACTIVITY MANAGEMENT

- After students are seated, explain the Explore to students. Ask if there are any questions. Then have one student distribute the pieces of rope to each group and another student distribute the meter sticks.
- Students often rush and don't measure accurately. Stress the importance of measuring carefully.
- Make sure students tie their knots about the same tightness; they should not have some tight and some loose.
- **Common Error** In Exercise 3, confirm that students list the ordered pairs with the number of knots as the x -coordinate.
- **Common Error** When students make a scatter plot in Exercise 4, some may connect the points. If this happens, ask the students what an x -value of 2.5 would mean. Probe why it doesn't make sense to have 2.5 knots.

Activity and Closure Questions

Ask these questions as a class.

1. Ask each group to state their y -intercept. Why are all the y -intercepts the same?

Answer: All the groups should have a y -intercept of 90 because everyone started with 90 cm of rope.

2. What value did your group use to represent the change in the length of the rope each time a knot was tied? How did you determine this value? Why aren't these values the same for every group?

Answer: Answers may vary. Students subtracted the new length of the rope from the former length after each knot was tied and estimated the average amount of change in the length of the rope. Each group should have similar values, but the tightness of the knots and inaccurate measuring are two factors that contribute to different values. Estimating the average also results in differences between the values.

3. Suppose you started with a piece of rope that was 300 centimeters in length and the average change in the length of the rope was 15 centimeters each time a knot was tied. Write and graph an equation that represents this situation.

Answer: $y = 300 - 15x$. Write and graph this equation on the board.

4. How does the average change in the length of the rope affect the graph? Why is the graph decreasing? How can you tell from the equation that the graph is decreasing?

Answer: The average change affects the steepness of the graph. Probe to help students determine that this is the *slope* of the graph. The graph is decreasing because the length of the rope decreases each time a knot is tied. You can tell the graph is decreasing from the equation because $15x$ is being subtracted from 300.

5. Suppose you had to graph the equation $y = 300 + 15x$. Would the graph increase or decrease? How do you know? What situation might this equation represent?

Answer: The graph would increase. You can tell the graph would increase from the equation because $15x$ is being added to 300. This could represent a number of situations. A possible situation is if you let x represent the number of knots untied and y represent the length of a piece of rope after a knot is untied. The starting length of the knotted rope is 300 centimeters. You gain 15 cm each time you untie a knot.

LESSON TRANSITION

Students will be writing more equations in this lesson. This activity will help them to think of the y -intercept of the equation as the initial value in real life situations and to think of the slope as the rate of change as they will do in Examples 4 and 5.